Combined Feedback Linearization and Second Order Sliding-Mode Control for a 2DOF Helicopter in the Presence of Disturbances and Uncertainties

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***Abstract*—This paper is concerned with the problem of tracking control for a multi-input multi-output (MIMO) twin rotor helicopter in the presence of uncertainties and disturbances. Two classes of robust controllers are constructed; the first is designed by combining the feedback linearization strategy with the second order sliding mode twisting algorithm technique, and the second is designed by using a new Lyapunov function approach, which can overcome the problem of chattering entirely. It is shown that the first designed controller can assure that all the signals remain bounded and the tracking error converges to zero in the best manner. Simulation results are presented to show the effectiveness of the proposed approaches.**

***Keywords*—second order sliding mode; feedback linearization; twin rotor helicopter; Lyapunov stability; chattering free, MIMO system.**

1. Introduction

Miniature unmanned vehicles (MUVs) are becoming popular due to their compact size, high maneuverability and high size-to-payload ratio. This is especially true with Vertical Takeoff and Landing (VTOL) vehicles due to their distinct capabilities to maneuver in any direction and to hover, even in highly confined areas [1]. From all classes of MUVs, miniature unmanned helicopters (MUHs) have advantages over fixed-wing UAVs because they take-off and land vertically, they do not require a runway, and they have the ability to hover and fly in low altitudes [2]. However, the control of MUHs is a challenging problem due to unknown nonlinearities and couplings in their dynamics that make it important to design robust nonlinear controllers. Since the number of inputs are less than degrees of freedom (DOF), MUHs are considered underactuated systems. Therefore, Robustness is one of the critical issues which must be considered in the control system design for such high-performance autonomous helicopter, since any mathematical helicopter model will unavoidably have uncertainty due to the empirical representation of aerodynamic forces and moments [3]. 2-DOF helicopter is considered as the prototype of a helicopter in pitch and yaw movement and hence it gained a lot of interests.

Rahideh, Shaeed, and Huijberts derived the general dynamic model of twin rotor multi-input–multi-output system (TRMS) with counterbalance weight using Newtonian and Lagrangian methods based analytical approaches and neural networks based empirical approaches [4]. TRMS is a 2DOF nonlinear system with cross couplings; it consists of a beam with two rotors connected at its ends which are driven by separate DC motors and the beam is counterbalanced by an arm having weight at its end [5]. These results were used by Yang and Hsu to design an adaptive nonlinear backstepping controller as well as by Pandey and Laxmi to design an optimal state feedback controller based on linear quadratic regulator (LQR) after the linearization of TRMS model [6-7]. Feedback linearization via static state feedback based on a linear model was designed by G. Mustafa and N. Iqbal to improve the effectiveness of angular position control of the system [8]. Xiuyan Wang has linearized the system and designed an adaptive linear sliding-mode Controller (LSMC) [9]. Sanjoy and Chitralekha considered an adaptive second order sliding mode control (SOSMC) method in which robustness is improved and input chattering is reduced compared to with the conventional controllers [10]. Here, the system was decoupled into two single-input–single-output (SISO) subsystems and the cross coupling was considered as uncertainties for each other since SMC is a good control strategy to robust systems in the presence of disturbances and uncertainties. In spite of its proven robustness, the SMC suffers from the inherent disadvantage of high-frequency oscillations of the control signal known as chattering [11]. This problem makes the implementation of SMC impossible for electromechanical systems as high-frequency oscillations can actuate unmodeled dynamics of the system and cause mechanical wear in it. However, evolution of SMCs of second as well as higher order conquered this problem of chattering to a large extent [12]. Some algorithms, which propose a solution to the above control problem, have been presented in [13]. Because the high order SMC requires complex calculations, a chattering free sliding mode control (CFSMC) based on adjusting the sliding condition can be obtained from Lyapunov stability theorem.

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Feedback linearization (FBL) is an approach to nonlinear control design in which a nonlinear state feedback control law is applied that, in principle, cancels all system nonlinearities. However, to perform FBL, the system nonlinearities must be completely known, including their derivatives up to some order depending on how they enter the dynamics. This is a potential problem in flight control since the aerodynamic forces and moments cannot be modeled precisely. To achieve robustness against such model errors, the combination of FBL and SOSMC (FLSOSMC) is proposed to augment the FBL controller [14].

In this work, a second-order sliding mode control scheme using the twisting algorithm combined with feedback linearization for the nonlinear TRMS without a counterbalance weight is proposed, in which the aim is the tracking of yaw and pitch angles, that is, position control. One of the prime interests of this paper is that the dynamics will not be decoupled and then the TRMS will be considered as a complete multiple-input-multiple-output (MIMO) system. Theoretical proof of the stability of the closed-loop system for SOSMC, FLSOSMC, and CFSMC is also addressed in the sense of Lyapunov. To confirm the effectiveness of the proposed control, results for computer simulations are also given.

The remainder of the paper is arranged as follows. Section 2 mathematically describes the dynamic model and the parameters of the system are specified. In Section 3, control laws based on FBL, SOSMC twisting algorithm (SOSMCTA), FLSOSMC, and CFSMC are designed to perform the tracking objective of pitch and yaw angles. The study of model stability is also carried out in this section. In section 4, several simulations of the model under disturbance for the case of trajectory tracking show the relevance of the proposed controller in comparison with linear quadratic regulator with integral (LQR+I), LSMC, FBL, SOSMCTA, and CFSMC controllers which are described in this work. Finally some conclusions are presented in section 5.

2. Dynamic Model

The TRMS mathematical model without a counterbalance weight can be derived from [4]. The helicopter system dynamic model is shown in Fig. 1. Consider L the distance from the motors (both pitch and yaw) to the pivot, R_{cm} the distance from the center of mass to the pivot, p the pitch angle relative to the horizontal axis, y the yaw angle (the reference for yaw is not relevant). The other parameters are shown in Table 1.

The dynamics of the pitch and yaw movement are mathematically expressed as follows

$$F_g = m_h g R_{cm} cos(p) \tag{1}$$

$$\delta = m_h R_{cm}^2 \sin p \cos p \, \dot{y}^2 \tag{2}$$

$$\tau = 2\dot{p}\dot{y} m_h R_{cm}^2 \cos p \sin y \tag{3}$$

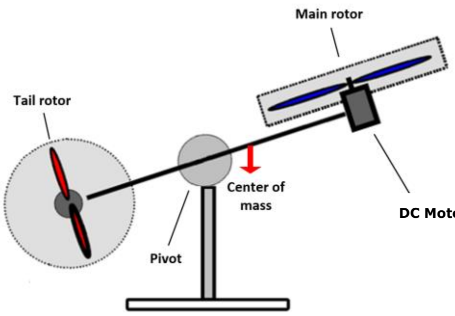

Figure 1. Aerodynamic model of the helicopter system.

Table 1. Dynamic model parameters

k_{pp} (k_{yy})	The force generated in the pitch (yaw) direction by a unit voltage applied to the pitch (yaw) motor
k_{yp} (k_{py})	The torque generated in the yaw (pitch) direction by a unit voltage applied to the pitch (yaw) motor
V_p (V_y)	Input voltage to pitch (yaw) motor
m_h	Total moving mass of 2DOF helicopter
J_{eqy} (J_{eqp})	Total moment of inertia about yaw (pitch) axis
c_p (c_y)	Equivalent viscous damping about pitch (yaw) axis
g	Gravitational constant
F_g	The force due to gravity acting through the center of mass (a gravitational disturbance)
$\boldsymbol{\tau}$	Yaw Position Dependent Torque

$$(J_{eqy} + m_h R_{cm}^2 \cos^2 p)\ddot{y} = k_{yy}V_y + k_{yp}V_p - c_y\dot{y} + \tau \tag{4}$$

$$(J_{eqp} + m_h R_{cm}^2)\ddot{p} = k_{pp}V_p - F_g + k_{py}V_y - c_p\dot{p} - \delta \tag{5}$$

Suppose the following expressions:

$$\underline{f}(x) = \begin{bmatrix} \frac{-F_g - c_p\dot{p} - \delta}{(J_{eqp}+m_h R_{cm}^2)} \\ \frac{-c_y\dot{y}+\tau}{(J_{eqy}+m_h R_{cm}^2 \cos^2 p)} \end{bmatrix} \tag{6}$$

$$\underline{b}(x) = \begin{bmatrix} \frac{k_{pp}}{(J_{eqp}+m_h R_{cm}^2)} & \frac{k_{py}}{(J_{eqp}+m_h R_{cm}^2)} \\ \frac{k_{yy}}{(J_{eqy}+m_h R_{cm}^2 \cos^2 p)} & \frac{k_{yp}}{(J_{eqy}+m_h R_{cm}^2 \cos^2 p)} \end{bmatrix} \tag{7}$$

$$\underline{u}(x) = \begin{bmatrix} V_p \\ V_y \end{bmatrix} \quad (8) \qquad \underline{x_1} = \begin{bmatrix} p \\ y \end{bmatrix} \quad and \quad \underline{x_2} = \underline{\dot{x}_1} = \begin{bmatrix} \dot{p} \\ \dot{y} \end{bmatrix} \tag{9}$$

As a result,

$$\underline{\dot{x}_1} = \underline{x_2} \qquad and \qquad \underline{\dot{x}_2} = \underline{f}(x) + \underline{b}(x)\underline{u}(x) \tag{10}$$

Assumption 1: $m_h R_{cm}^2 \cos^2 p << J_{eqy}$.Thus, the term $m_h R_{cm}^2 \cos^2 p$ is negligible, especially if the center of mass is located at the pivot.

Assumption 2: the matrix $\underline{b}(x)$ is invertible.

3. CONTROL DESIGN

3.1 Control Objective

The primary flight control objective is to design the two control inputs u = $[V_p \; V_y]^T$ for the 2DOF MUH to track a reference altitude which is denoted by $\underline{x_d} = [p_r \; y_r]^T$, where p_r and y_r are the desired pitch and yaw angles, respectively, in the presence of model parametric uncertainties and external disturbances, while keeping the stability of the closed-loop dynamics. Mathematically, the objective is translated to finding appropriate input voltages V_p and V_y, such that $p \rightarrow p_r$ and $y \rightarrow y_r$ as $t \rightarrow \infty$.

Theorem 1: the system (10) is controllable.

Proof 1: Redefine the system (10) as

$$\underline{\dot{x}} = \underline{M}(x) + \underline{g}\underline{u}(x) \;\; where \;\; \underline{x} = \begin{bmatrix} p \\ \dot{p} \\ y \\ \dot{y} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{11}$$

With

$$\underline{M}(x) = \begin{bmatrix} x_2 \\ \frac{-m_h g R_{cm} cos(x_1) - c_p x_2 - m_h R_{cm}^2 \sin x_1 \cos x_1 {x_4}^2}{(J_{eqp} + m_h R_{cm}^2)} \\ x_3 \\ \frac{-c_y x_4 + 2 x_2 x_4 m_h R_{cm}^2 \cos x_1 \sin x_3}{J_{eqy}} \end{bmatrix} \tag{12}$$

$$and \qquad \underline{g} = \begin{bmatrix} 0 & 0 \\ \frac{k_{pp}}{(J_{eqp} + m_h R_{cm}^2)} & \frac{k_{py}}{(J_{eqp} + m_h R_{cm}^2)} \\ 0 & 0 \\ \frac{k_{yy}}{J_{eqy}} & \frac{k_{yp}}{J_{eqy}} \end{bmatrix} \tag{13}$$

Then consider the first two terms of the generalized controllability matrix $[g \quad ad_{M'} g \quad ad_{M^2} g \quad ad_{M^3} g]$

$$[g \quad ad_M g] = \begin{bmatrix} 0 & 0 \\ \frac{k_{pp}}{(J_{eqp} + m_h R_{cm}^2)} & \frac{k_{py}}{(J_{eqp} + m_h R_{cm}^2)} \\ 0 & 0 \\ \frac{k_{yy}}{J_{eqy}} & \frac{k_{yp}}{J_{eqy}} \end{bmatrix} \cdots$$

$$\begin{matrix} \frac{-k_{pp}}{(J_{eqp} + m_h R_{cm}^2)} & \frac{-k_{py}}{(J_{eqp} + m_h R_{cm}^2)} \\ \frac{c_p k_{pp}}{(J_{eqp} + m_h R_{cm}^2)} + \frac{2\delta}{x_4} \frac{k_{yy}}{J_{eqy}} & \frac{c_p k_{py}}{(J_{eqp} + m_h R_{cm}^2)} + \frac{2\delta}{x_4} \frac{k_{yp}}{J_{eqy}} \\ \frac{-k_{yy}}{J_{eqy}} & \frac{-k_{yp}}{J_{eqy}} \\ \frac{-\tau}{x_2} \frac{k_{pp}}{(J_{eqp} + m_h R_{cm}^2)} + c_y \frac{k_{yy}}{J_{eqy}} - \frac{k_{yy}}{J_{eqy}} \frac{\tau}{x_4} & \frac{-\tau}{x_2} \frac{k_{py}}{(J_{eqp} + m_h R_{cm}^2)} + c_y \frac{k_{yp}}{J_{eqy}} - \frac{k_{yp}}{J_{eqy}} \frac{\tau}{x_4} \end{matrix} \Bigg] \tag{14}$$

Thus

$$Rank([g \quad ad_M g]) = 4$$

which is equal to the system dimension. Then the theorem 1 is proved.

3.2 Control Design

3.2.1 FEEDBACK LINEARIZATION

The main idea of FBL strategy is to modify the system structure, so that the closed-loop control system transformed into a fully or partly linear one in which linear control techniques can be applied.

For the dynamic model in (10), the relative degrees of the system are {2, 2}, that is, the sum of relative degrees r = 4, which is equal to the system dimension. Therefore, the model is fully feedback linearizable.

Consider the certain helicopter system of (10) and let $\underline{y} = \underline{x_1}$ be the angle output vector of TRMS, and let the control law be defined by

$$\underline{u}(x) = \underline{b}^{-1}(x)\left(-\underline{f}(x) + \underline{\ddot{x}}_d - \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_1 \end{bmatrix} \underline{\dot{e}} - \begin{bmatrix} \lambda_2 & 0 \\ 0 & \lambda_2 \end{bmatrix} \underline{e}\right) \tag{15}$$

where $\underline{x_d} = \begin{bmatrix} p_r \\ y_r \end{bmatrix}$ and $\underline{e} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \underline{x_1} - \underline{x_d} = \begin{bmatrix} p - p_r \\ y - y_r \end{bmatrix}$, then the error dynamics, that is, the closed-loop system will be

$$\underline{\ddot{e}} + \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_1 \end{bmatrix} \underline{\dot{e}} + \begin{bmatrix} \lambda_2 & 0 \\ 0 & \lambda_2 \end{bmatrix} \underline{e} = 0 \tag{16}$$

The constants λ_1 and λ_2 have to be designed to place the eigenvalues of the above system in the left half plane to guarantee both the convergence to zero of error function and the overall stability of the system.

3.2.2 SECOND-ORDER SLIDING MODE CONTROL TWISTING ALGORITHM (SOSMC)

Twisting algorithm which is applicable to systems of relative degree 2, ensures that the sliding surface S(x) as well as its first derivative will converge to zero in finite time leading to a smooth control action.

In this section, the MIMO nonlinear system with the presence of uncertainties and external disturbance is described. Let the system (10) redefined as

$$\underline{\dot{x}_2} = \underline{f}(x) + \underline{\Delta f}(x,t) + \left(\underline{b}(x) + \underline{\Delta b}(x)\right)\underline{u}(x) + \underline{d}(t) \quad (17)$$

Where $\underline{\Delta f}(x,t)$ and $\underline{\Delta b}(x)$ are uncertain terms, and $\underline{d}(t)$ is the disturbance term. In practical system, uncertainties and disturbances are unknown and we have to assume upper bounds for them.

Assumption 3: the uncertain and disturbance terms are bounded by

$$\begin{cases} \left|\underline{\Delta f}(x,t)\right| \le \alpha \\ \left|\underline{\Delta b}(x)\right| \le \beta \\ \left|\underline{d}(t)\right| \le \gamma \end{cases} \quad (18)$$

where $\alpha, \beta, and\ \gamma$ are positive and unknown.

Let us choose two sliding manifolds as

$$\underline{s} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \underline{e} + \Gamma \int \underline{e}\, dt \rightarrow \underline{\dot{s}} = \dot{e} + \Gamma \underline{e} \quad (19)$$

where $\Gamma = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$ and To ensure $\lim_{t\to\infty} \underline{e} = \underline{0}$ the $\lambda > 0$ is chosen.The main idea behind the SOSM is to act on the second second-order derivative of the sliding variable s(x) rather than the first derivative as in standard-sliding modes [10]. The second derivative of s(x) is obtained as

$$\underline{\ddot{s}} = \underline{\ddot{e}} + \Gamma \dot{e} = \underline{\ddot{x}} - \underline{\ddot{x}_d} + \Gamma\left(\underline{\dot{x}} - \underline{\dot{x}_d}\right) \quad (20)$$

from (17) we have

$$\underline{\ddot{s}} = \underline{f}(x) + \underline{\Delta f}(x,t) + \left(\underline{b}(x) + \underline{\Delta b}(x)\right)\underline{u}(x) + \underline{d}(t) - \underline{\ddot{x}_d} + \Gamma \underline{\dot{e}} \quad (21)$$

Set $\underline{\ddot{s}} = 0$ to obtain the control law

$$\underline{u}(x) = \underline{b}^{-1}(x)\left(-\underline{f}(x) + \underline{\ddot{x}_d} - \Gamma \underline{\dot{e}} + \hat{u}\right) \quad (22)$$

The twisting controller can be defined as (Levant, 1993) [15]

$$\hat{u} = -M_0 sign(s) - M_1 sign(\dot{s}) \quad (23)$$

where $M_0, and\ M_1$ are positive values to be designed.

Theorem 2: Consider the uncertain system defined by (17) and let the control law be defined by

$$\underline{u}(x) = \underline{b}^{-1}(x)\left(-\underline{f}(x) + \underline{\ddot{x}_d} - \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_1 \end{bmatrix}\underline{\dot{e}} - M_0 sign(s) - M_1 sign(\dot{s})\right) \quad (24)$$

with

$$\begin{cases} |\Delta b(x)u(x)| \le \rho \\ M_1 > \alpha + \rho + \gamma \\ M_0 > M_1 + \alpha + \rho + \gamma \end{cases} \quad (25)$$

where ρ is positive and unknown, and if λ is small, we can assume that $sign(\dot{e}) = sign(\dot{s})$.Then, the closed loop system satisfies the sliding condition and all sliding surfaces and their derivatives converge to zero.

Proof 2: We consider the Lyapunov function as the square root of energy

$$V = \sum_{i=1}^{2} 2\sqrt{\frac{\dot{s}^2}{2} + M_0|s|} \quad (26)$$

the derivative of V is:

$$\dot{V} = \sum_{i=1}^{2} \frac{\dot{s}_i \ddot{s}_i + M_0 \dot{|s_i|}}{\sqrt{\dot{s}_i^{\,2}\left(\frac{1}{2} + M_0 \frac{|s_i|}{\dot{s}_i^{\,2}}\right)}} \quad (27)$$

using (21), the expression of the derivative for the surface i become

$$\dot{V}_i = \frac{\dot{s}\left(f(x) + \Delta f(x) + \left(\underline{b}(x) + \Delta b(x)\right)\underline{u}(x) + d(t) - \ddot{x}_d + \lambda \dot{e}\right) + M_0 sign(s)\dot{s}}{|\dot{s}|\sqrt{\frac{1}{2} + M_0 \frac{|s|}{\dot{s}^2}}}$$

Using the control law (24), we have

$$\dot{V}_i = \frac{\dot{s}\left(f(x) + \Delta f(x,t) + \left(\underline{b}(x) + \Delta b(x)\right)\left(\underline{b}^{-1}(x)\left(-f(x) + \ddot{x}_d - \lambda_1 \dot{e} - M_0 sign(s) - M_1 sign(\dot{s})\right)\right) + d(t) - \ddot{x}_d + \lambda \dot{e}\right) + M_0 sign(s)\dot{s}}{|\dot{s}|\sqrt{\frac{1}{2} + M_0 \frac{|s|}{\dot{s}^2}}}$$

$$= \frac{\dot{s}(f(x) + \Delta f(x,t) - f(x) + \ddot{x}_d - \lambda_1 \dot{e} - M_0 sign(s) - M_1 sign(\dot{s}) + \Delta b(x)u(x) + d(t) - \ddot{x}_d + \lambda \dot{e}) + M_0 sign(s)\dot{s}}{|\dot{s}|\sqrt{\frac{1}{2} + M_0 \frac{|s|}{\dot{s}^2}}}$$

$$= \frac{\dot{s}(\Delta f(x,t) - M_1 sign(\dot{s}) + \Delta b(x)u(x) + d(t) + (\lambda - \lambda_1)\dot{e})}{|\dot{s}|\sqrt{\frac{1}{2} + M_0 \frac{|s|}{\dot{s}^2}}}$$

$$= \frac{sign(\dot{s})(\Delta f(x,t) - M_1 sign(\dot{s}) + \Delta b(x)u(x) + d(t) + (\lambda - \lambda_1)\dot{e})}{\sqrt{\frac{1}{2} + M_0 \frac{|s|}{\dot{s}^2}}}$$

$$= \frac{sign(\dot{s})(\Delta f(x,t) - M_1 sign(\dot{s}) + \Delta b(x)u(x) + d(t) + (\lambda - \lambda_1)\dot{e})}{\sqrt{\frac{1}{2} + M_0 \frac{|s|}{\dot{s}^2}}}$$

$$= \frac{sign(\dot{s})(\Delta f(x,t) + \Delta b(x)u(x) + d(t) + (\lambda - \lambda_1)\dot{e}) - M_1}{\sqrt{\frac{1}{2} + M_0 \frac{|s|}{\dot{s}^2}}}$$

$$\dot{V} \le \frac{|\Delta f(x,t)| + |\Delta b(x)u(x)| + |d(t)| + (\lambda - \lambda_1)|\dot{e}| - M_1}{\sqrt{\frac{1}{2} + M_0 \frac{|s|}{\dot{s}^2}}} \quad (28)$$

From (25), and if we choose $\lambda_1 > \lambda$, we conclude that the derivative of Lyapunov function is negative

$$\dot{V} \le -\frac{M'}{\sqrt{\frac{1}{2} + M_0 \frac{|s|}{\dot{s}^2}}} < 0 \quad M' = M_1 - (\alpha + \rho + \gamma + (\lambda_1 - \lambda)|\dot{e}|) > 0 \quad (29)$$

It is obvious that the Lyapunov derivative for the system (10) will be

$$\dot{V} = \frac{-M_1+(\lambda-\lambda_1)|\dot{e}|}{\sqrt{\frac{1}{2}+M_0\frac{|s|}{\dot{s}^2}}} < 0 \quad (30)$$

and as a result, the Lyapunov function converges to zero, and so s and $\dot{s}$ tend to zero. Thus the proof is achieved completely.

3.2.3 Combined Feedback Linearization with Second-Order Sliding Mode Control (FLSOSMC)

Reconsider the system (17) with the control law (15) combined with (24).

Theorem 3: Consider the uncertain system defined by (17) and let the control law be defined by

$$\underline{u}(x) = \underline{b}^{-1}(x)\left(-\underline{f}(x) + \underline{\ddot{x}}_d - \begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_1\end{bmatrix}\underline{\dot{e}} - \begin{bmatrix}\lambda_2 & 0\\ 0 & \lambda_2\end{bmatrix}\underline{e} - M_0 sign(s) - M_1 sign(\dot{s})\right) \quad (31)$$

with

$$|\dot{s}| \geq |\dot{e}| \quad (32)$$

Then, the closed loop system satisfies the sliding condition and all sliding surfaces and their derivatives converge to zero.

Proof 3: We consider the Lyapunov function

$$V = M_0|s| + \dot{s}^T\dot{s} = \sum_{i=1}^{2} M_0|s_i| + \dot{s_i}^2 \quad (33)$$

the derivative of V for the pitch or yaw subsystem is

$$\dot{V} = M_0 sign(s)\dot{s} + \dot{s}\big(\Delta f(x,t) - M_1 sign(\dot{s}) + \Delta b(x)u(x) + d(t) + (\lambda-\lambda_1)\dot{e} - \lambda_2 e - M_0 sign(s)\big)$$

$$= \dot{s}(\Delta f(x,t) - M_1 sign(\dot{s}) + \Delta b(x)u(x) + d(t) + (\lambda-\lambda_1)\dot{e} - \lambda_2 e)$$

From (19)

$$= \dot{s}\big(\Delta f(x,t) + \Delta b(x)u(x) + d(t)\big) - \lambda_2\left(\frac{\dot{s}-\dot{e}}{\lambda}\right)\dot{s} - M_1|\dot{s}| + (\lambda-\lambda_1)|\dot{e}||\dot{s}|$$

$$= \dot{s}\big(\Delta f(x,t) + \Delta b(x)u(x) + d(t)\big) - \frac{\lambda_2}{\lambda}\dot{s}^2 + \frac{\lambda_2}{\lambda}|\dot{e}||\dot{s}| - M_1|\dot{s}| + (\lambda-\lambda_1)|\dot{e}||\dot{s}|$$

So

$$\dot{V} \leq |\dot{s}|\left(|\Delta f(x,t)| + |\Delta b(x)u(x)| + |d(t)| + \left(\lambda-\lambda_1+\frac{\lambda_2}{\lambda}\right)|\dot{e}| - \frac{\lambda_2}{\lambda}|\dot{s}| - M_1\right)$$

$$\dot{V} \leq |\dot{s}|\left(|\Delta f(x,t)| + |\Delta b(x)u(x)| + |d(t)| + \left(\lambda-\lambda_1+\frac{\lambda_2}{\lambda}\right)|\dot{e}| - \frac{\lambda_2}{\lambda}|\dot{s}| - M_1\right)$$

$$\dot{V} \leq |\dot{s}|\left(\alpha+\beta+\gamma+\left(\lambda-\lambda_1+\frac{\lambda_2}{\lambda}\right)|\dot{e}| - \frac{\lambda_2}{\lambda}|\dot{s}| - M_1\right) \quad (34)$$

From (32), we conclude that the derivative of Lyapunov function is negative

$$\dot{V} \leq -|\dot{s}|M'' < 0 \quad M'' = M' + \frac{\lambda_2}{\lambda}(|\dot{s}| - |\dot{e}|) > 0 \quad (35)$$

It is obvious that the Lyapunov derivative for the system (10) will be

$$\dot{V} \leq |\dot{s}|\left(\left(\lambda-\lambda_1+\frac{\lambda_2}{\lambda}\right)|\dot{e}| - \frac{\lambda_2}{\lambda}|\dot{s}| - M_1\right) < 0 \quad (36)$$

As a result, the Lyapunov function converges to zero, and so s and $\dot{s}$ tend to zero. Thus the proof is achieved completely.

Remark 1. A robust real time differentiator time has been added to the controller in order to estimate s and its derivative. Real-time differentiation is an old and well-studied problem. Exact derivatives may be calculated by successive implementation of a robust exact first-order differentiator (Levant 1998 a) with finite-time convergence. That differentiator is based on 2-sliding mode and is proved to feature the best possible asymptotics in the presence of infinitesimal Lebesgue-measurable measurement noises, if the second time derivative of the unknown base signal is bounded [16]. Therefore, a special differentiator is to be designed as

$$\dot{z}_0 = -\gamma_1 L^{\frac{1}{2}}\sqrt{|z_0 - s|}\,sign(z_0 - s) + z_1 \quad (37)$$

$$\dot{z}_1 = -\gamma_2 L^{\frac{1}{2}}\sqrt{|z_0 - s|}\,sign(z_1 - \dot{z}_0) \quad (38)$$

Where z_0 and z_1 are real time estimations of s and $\dot{s}$, respectively. Differentiator parameters γ_1 and γ_2 are assumed to be 1.1 and 1.5, respectively, and L is selected experimentally. Using the estimators of sliding surface and its derivative would, in principal, solve the chattering problem, but the behavior of states will be affected negatively.

3.3 Alternative Chattering Free SMC (CFSMC)

To remove chattering of SMC, we use the Lyapunov approach. Let us choose the Lyapunov function

$$V = \frac{1}{2}\underline{\sigma}^T\underline{\sigma} = \frac{1}{2}\sum_{i=1}^{2}\sigma_i^2 \quad (39)$$

where

$$\underline{\sigma} = \begin{bmatrix}\sigma_1\\ \sigma_2\end{bmatrix} = \begin{bmatrix}\dot{e}_1 + \xi e_1\\ \dot{e}_2 + \xi e_2\end{bmatrix} \text{ with } \xi \text{ is a positive constant} \quad (40)$$

are the conventional sliding surfaces [17]. To make the time derivative of (39) negative definite, we have to find an appropriate control input that must be satisfied the following inequality which is a new version of sliding condition of [17]

$$\dot{V}_i = \sigma\dot{\sigma} < -\eta\sigma^2 \quad (41)$$

where η is a strictly positive constant.

Remark 2. In this design method, the right side term of (41) is different from that of conventional SMC.

Theorem 4: Consider the uncertain system defined by (17) then the control law be defined by

$$\underline{u}(x) = \underline{b}^{-1}(x)\left(-\underline{f}(x) + \underline{\ddot{x}}_d - \begin{bmatrix} \xi & 0 \\ 0 & \xi \end{bmatrix}\underline{\dot{e}} - k\sigma\right) \quad (42)$$

With

$$k = \vartheta + \frac{1}{\sigma}(\alpha + \beta + \gamma) \quad (43)$$

where ϑ is a positive constant, satisfies the condition (41).

Proof 4.

Using (4), The derivative for pitch or yaw subsystem is

$$\dot{V} = \sigma\dot{\sigma} = \sigma(\ddot{e} + \xi\dot{e}) \quad (44)$$

$$= \sigma\left(f(x) + \Delta f(x) + \left(\underline{b}(x) + \Delta b(x)\right)\underline{u}(x) + d(t) - \ddot{x}_d + \xi\dot{e}\right)$$

from (42)

$$\dot{V} = \sigma\left(f(x) + \Delta f(x) + \left(\underline{b}(x) + \Delta b(x)\right)\underline{b}^{-1}(x)(-f(x) + \ddot{x}_d - \xi\dot{e} - k\sigma) + d(t) - \ddot{x}_d + \xi\dot{e}\right)$$

$$= \sigma(\Delta f(x) + \Delta b(x)u(x) - k\sigma + d(t))$$

from (41)

$$\Delta f(x) + \Delta b(x)u(x) - k\sigma + d(t) \leq -\eta\sigma$$

then

$$k \geq \eta + \frac{\Delta f(x) + \Delta b(x)u(x) + d(t)}{\sigma}$$

$$k \geq \eta + \frac{|\Delta f(x)| + |\Delta b(x)u(x)| + |d(t)|}{\sigma}$$

$$k \geq \eta + \frac{\alpha + \beta + \gamma}{\sigma}$$

take

$$k = \vartheta + \frac{1}{\sigma}(\alpha + \beta + \gamma)$$

With $\vartheta > \eta$. Therefore, (42) with (43) satisfies (41).

Remark 3. The control input is finite and available when s is equal to zero, because the denominator of (43) will be removed when multiplied by s in (42).

4. COMPUTER SIMULATIONS

In this section, computer simulations were performed to demonstrate the feasibility of the proposed controller. The system parameters selected for simulations are the same as [9]. The other parameters are shown in table 2. To demonstrate the effectiveness of the proposed methodology and to make a comparative study between conventional control techniques and the proposed control scheme, FLSOSMC was compared to LSMC, SOSMCTA, FBL, CFSMC, and LQR+I. In addition, the system is subjected to uncertainties and disturbance signals.

Reference values for tracking pitch and yaw are equal to 1 and 3 rad, respectively. Parameters of controllers are as follows

$$\lambda = 0.001, \lambda_2 = 10, \lambda_1 = 15, L = 5, M_1 = 0.5, M_0 = 4$$

$$\xi = 1, \alpha = 0.1, \beta = 0.05, \gamma = 0.1, \vartheta = 40$$

The disturbance and uncertain terms are

$$\underline{d}(t) = \begin{bmatrix} 3\sin(3t) \\ 3\cos(5t) \end{bmatrix} \; and \; \underline{\Delta f}(x,t) = \begin{bmatrix} 0.01\cos(p) \\ 0.2\cos(5t) \end{bmatrix} \quad (45)$$

Table 2. Parameters for simulations.

R_{cm}	0.05	m
c_p	0.6	N.m.s/rad
c_y	0.318	N.m.s/rad
m_h	0.7	kg
g	9.81	m/s^2

A. Tracking Problem: Step Response

The results of various controllers for the system (10) which has no uncertainties and disturbances are given in Figs. 2 and 3. Figs. 4 and 5 show the step responses of the system (17). Better performance of FLSOSMC is clearly obvious in either case.

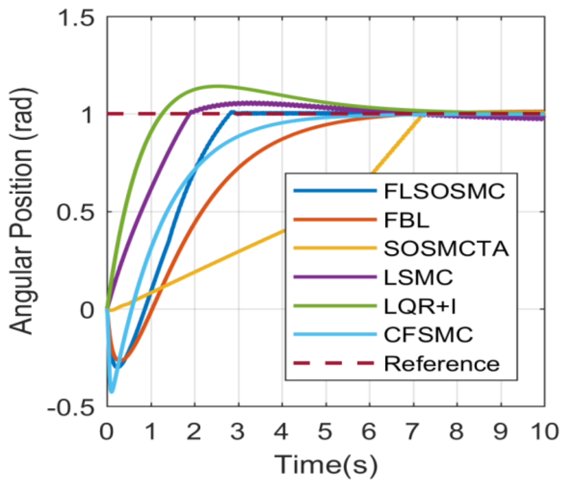

Figure 2: Pitch response of the system (10)

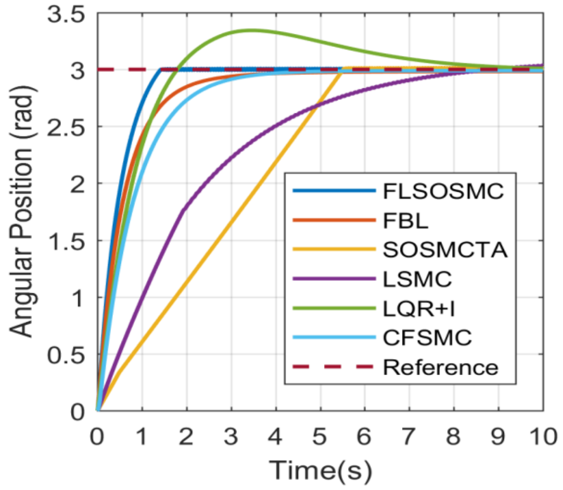

Figure 3: Yaw response of the system (10)

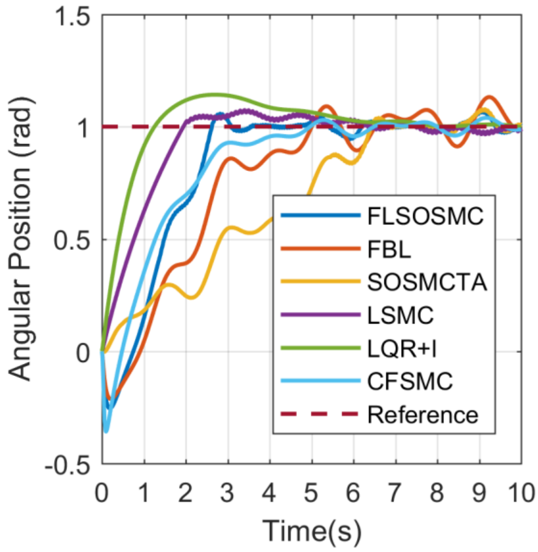

Figure 4: Pitch response of the system (17)

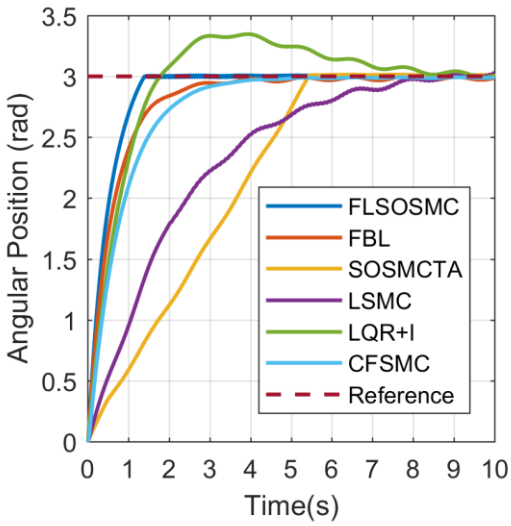

Figure 5: Yaw response of the system (17)

We can notice that in the presence or absence of disturbances and uncertainties, step responses of FLSOSMC are fastest and more robust as well as have excellent convergence and robustness properties. The input voltages for pitch and yaw motors are shown in Fig. 6. It can be seen that the chattering problem of traditional SMC is solved to a certain extent and the control inputs became smoother.

B. Tracking Problem: Sinusoidal Reference

Moreover, to prove the superiority of the proposed controller, the tracking performance for the helicopter system with a varying time reference trajectory is also presented. Reference values for tracking pitch and yaw are

$$\underline{x_d}(t) = \begin{bmatrix} \sin\left(\frac{1}{2\pi}t\right) \\ 0.5\sin\left(\frac{1}{2\pi}t\right) \end{bmatrix}$$

The results for the system (17) are shown in Figs. 7 and 8.

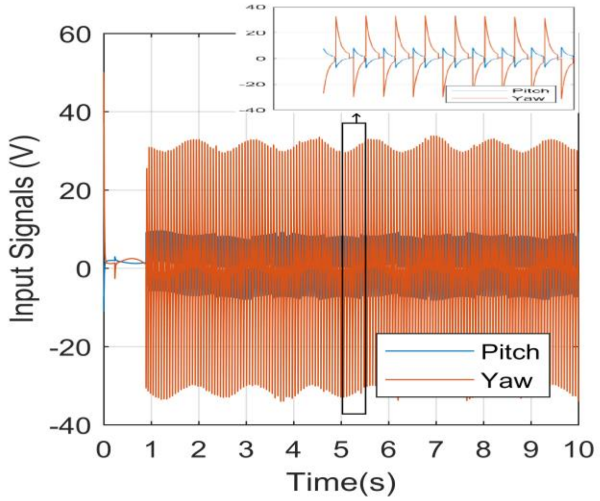

Figure 6: Input signals of FLSOSMC for the system (17)

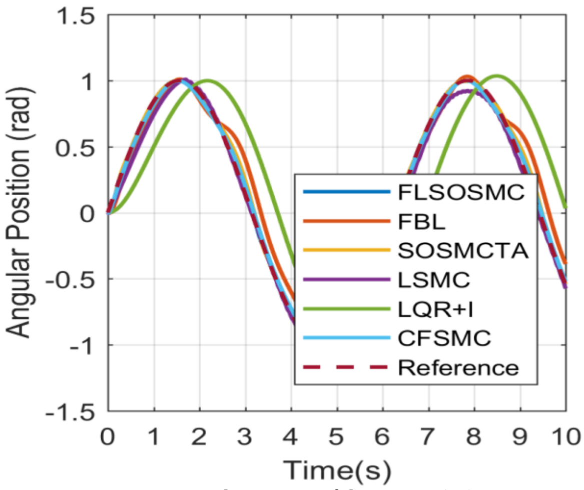

Figure 7: Pitch response of the system (17)

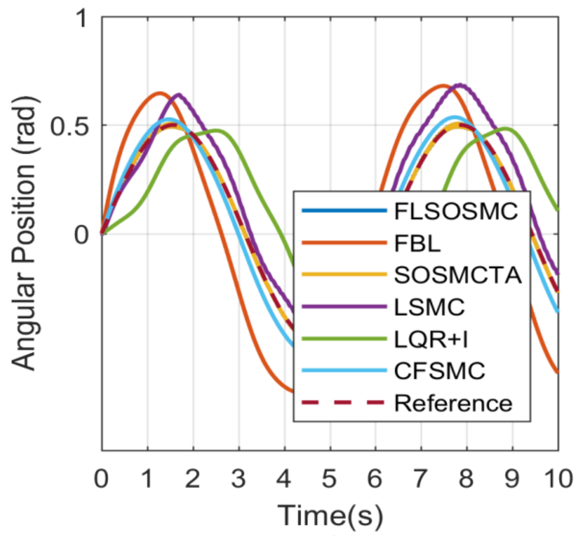

Figure 8: Yaw response of the system (17)

By comparing the results with respect to robustness and convergence, it can be concluded that FLSOSMC and SOSMCTA controllers are better than other controllers.

5. CONCLUSION

In this paper, control problem of an uncertain MIMO nonlinear system with cross-coupling effect is developed via FLSOSMC. Compared with the existing controllers, FLSOSMC strategy has less tracking errors, and therefore more accuracy, less chattering, and less insensitivity to uncertainties and external disturbances.

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