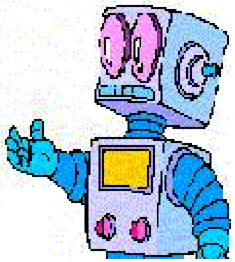
INTRODUCTION TO ROBOTICS

(Kinematics, Dynamics, and Design)

SESSION # 16:

MANIPULATOR DYNAMICS



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Manipulator Dynamics

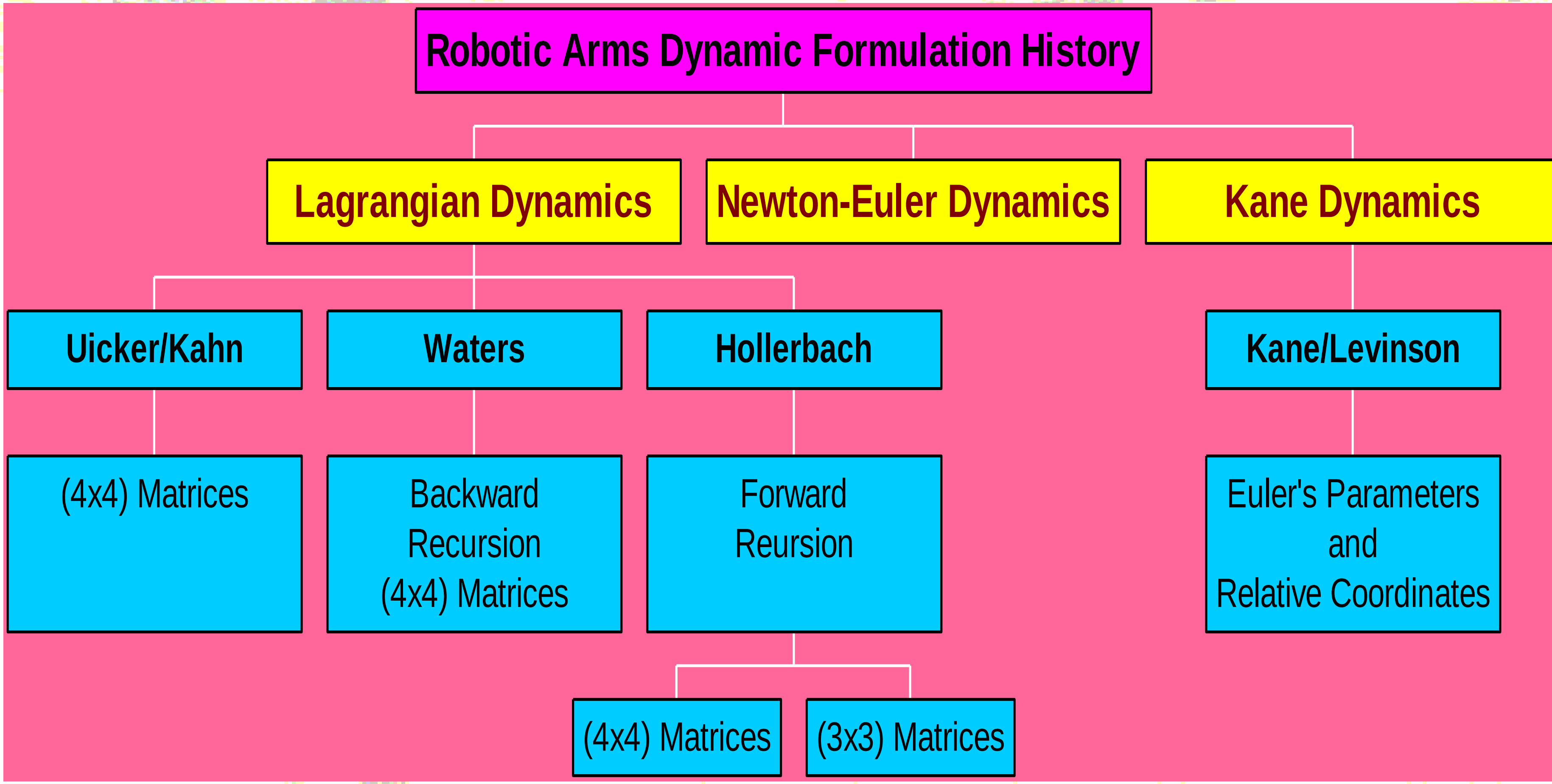
- So far we have only studied motion of manipulators without regard to forces causing the motion.

- Let us now derive the equations of motion for manipulator arms. In dynamics, we generally consider the following issues:

  - **Forward Dynamics:** Computing the resulting motion of the manipulator arm ($\theta, \dot{\theta}, \ddot{\theta}$) under the application of a set joint torques (τ). This is useful for <u>simulation</u> of the arm.

  - **Inverse Dynamics:** Computing the vector of joint torques (τ) for the given joint motion trajectory ($\theta, \dot{\theta}, \ddot{\theta}$) . This is useful for <u>controlling</u> of the arm.

Manipulator Dynamics



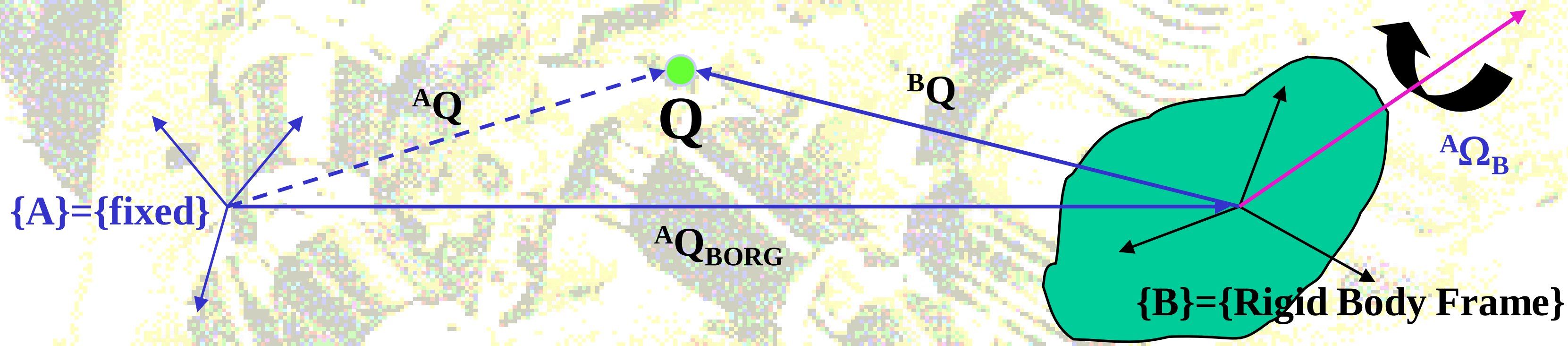
Manipulator Dynamics

Author	Method	Multiplications	Additions
Uicker/Kahn (Lagrangian Dyn.)	(4 × 4) Matrices	**66,271**	**51,548**
Waters (Lagrangian Dyn.)	(4 × 4) Backward Recursion	**7,051**	**5,652**
Hollerbach (Lagrangian Dyn.)	(4 × 4) Forward Recursion	**4,388**	**3,586**
Hollerbach (Lagrangian Dyn.)	(3 × 3) Forward Recursion	**2,195**	**1,719**
Newton-Euler	Recursive	**852**	**738**
Kane/Levinson	Kane Dynamics	**646**	**394**
Raibert/Horn	Configuration Space Method (CSM)	**468**	**264**
Yang/Tzeng	Dyn. Simplification by Design	**72**	**34 + 4 Trig. Functions.**

Manipulator Dynamics

➢ Linear Accelerations of Rigid Bodies:

Consider a point "Q" in space, and describe its kinematics in two frames {A} and {B}.



From Chapter-5 we have:

$$^{A}Q \equiv {}^{A}Q_{BORG} + {}^{A}_{B}R\,{}^{B}Q$$

$$^{A}V_{Q} = {}^{A}V_{BORG} + {}^{A}\Omega_{B} \times {}^{A}_{B}R\,{}^{B}Q + {}^{A}_{B}R\,{}^{B}V_{Q}$$

Differentiating the velocity equation with respect to the time we have:

Manipulator Dynamics

- **Linear Accelerations of Rigid Bodies:**

Noting that: $${}^{A}_{B}\dot{R} = {}^{A}\Omega_{B} \times {}^{A}_{B}R$$

$${}^{A}\dot{V}_{Q} = {}^{A}\dot{V}_{BORG} + {}^{A}_{B}R\,{}^{B}\dot{V}_{Q} + 2\,{}^{A}\Omega_{B} \times {}^{A}_{B}R\,{}^{B}V_{Q} + \\ + {}^{A}\dot{\Omega}_{B} \times {}^{A}_{B}R\,{}^{B}Q + {}^{A}\Omega_{B} \times ({}^{A}\Omega_{B} \times {}^{A}_{B}R\,{}^{B}Q)$$

If ^{B}Q is constant (on the R.B.), then: $${}^{B}V_{Q} = {}^{B}\dot{V}_{Q} = 0$$

$${}^{A}\dot{V}_{Q} = {}^{A}\dot{V}_{BORG} + {}^{A}\dot{\Omega}_{B} \times {}^{A}_{B}R\,{}^{B}Q + {}^{A}\Omega_{B} \times ({}^{A}\Omega_{B} \times {}^{A}_{B}R\,{}^{B}Q)$$

Manipulator Dynamics

➢ Angular Acceleration of Rigid Bodies:

Consider:

- Frame {B} rotating relative to {A} with: $^{A}\Omega_{B}$

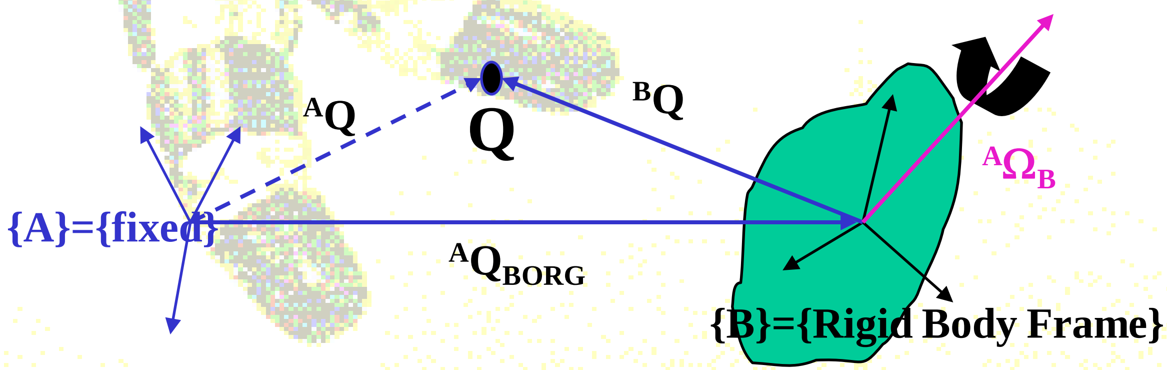
- Frames {C} rotating relative to {B} with: $^{B}\Omega_{C}$

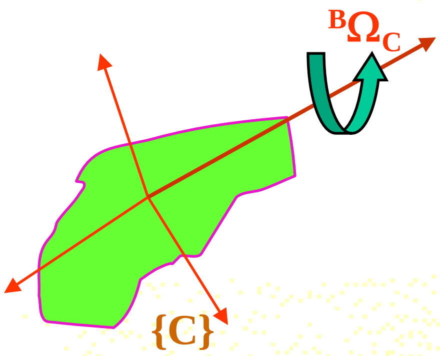
Then:

$$^{A}\Omega_{C} \equiv {}^{A}\Omega_{B} + {}^{A}_{B}R\,{}^{B}\Omega_{C}$$

Sum the vectors in frame {A}

$$^{A}\dot{\Omega}_{C} = {}^{A}\dot{\Omega}_{B} + {}^{A}_{B}R\,{}^{B}\dot{\Omega}_{C} + {}^{A}\Omega_{B} \times {}^{A}_{B}R\,{}^{B}\Omega_{C}$$





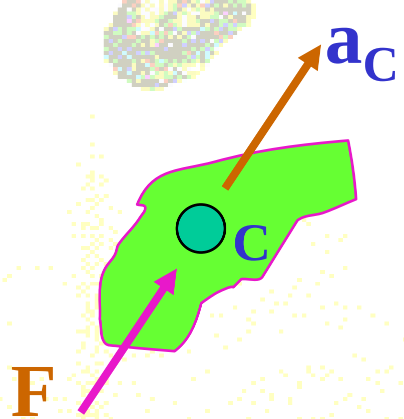
Manipulator Dynamics

- **Newtonian Mechanics:**

For a Rigid Body whose center of mass is accelerating with "a_C", the Force "F" acting at the mass center is given by:

The Newton's Law of Motion:

$$F = \sum f_i = \dot{P} = m\dot{v}_C = ma_C$$ **= (Time rate of change of momentum)**



Manipulator Dynamics

- **Newtonian Mechanics:**

For a Rigid Body rotating with an angular velocity "ω", and an angular accelerating "α", the Moment "N" which must be acting on the body to cause this motion, is given by:

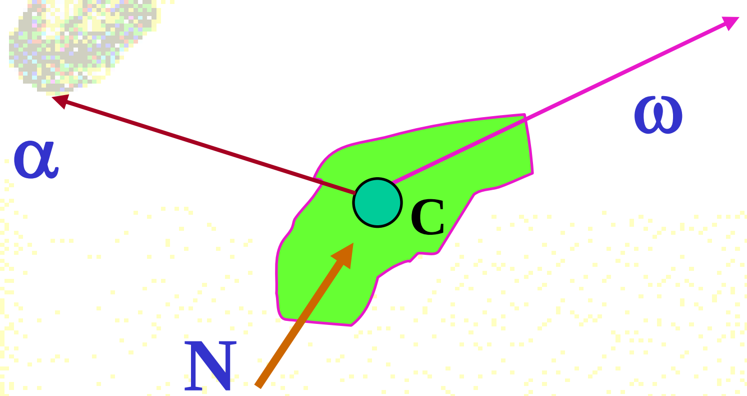
The Euler's Equation:

$$N = {}^{C}I \cdot \alpha + \omega \times ({}^{C}I \cdot \omega)$$

(The rotational analogy of the Newton's 2nd law comes from the Principle of Moment of Momentum)

where:

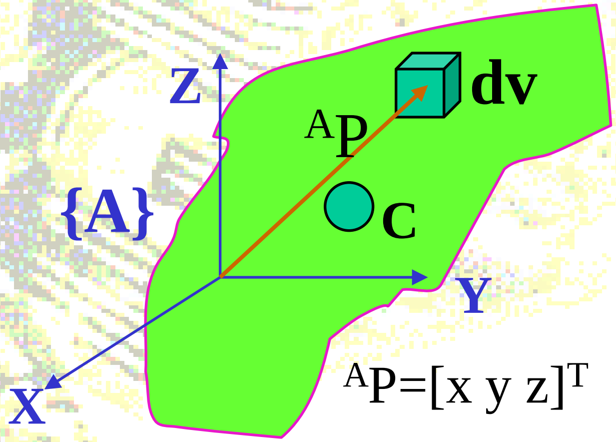
$^{C}\mathbf{I}$ **= Inertia Tensor of the R.B. written in frame {C}**



Manipulator Dynamics

- **Mass Distribution:** **The Inertia Tensor of an object describes the object's mass distribution (a generalization of the scalar moment of inertia). Relative to a frame {A} is expressed as:**

$$ {}^{A}I = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} $$



where:

$$ I_{xx} = \iiint_v (y^2 + z^2)\rho dv; \quad I_{xy} = \iiint_v xy\rho dv $$

$$ I_{yy} = \iiint_v (x^2 + z^2)\rho dv; \quad I_{xz} = \iiint_v xz\rho dv $$

$$ I_{zz} = \iiint_v (x^2 + y^2)\rho dv; \quad I_{yz} = \iiint_v yz\rho dv $$

Manipulator Dynamics

- **Iterative Newton-Euler Dynamic Formulation:**

  Let us now study the problem of computing the vector of joint torques (τ) for the given joint motion trajectory ($\theta, \dot{\theta}, \ddot{\theta}$). (The Inverse Dynamics problem useful for controlling of the arm).

- **Outward Iterations to Compute Velocities and Accelerations:**

  To study dynamics from Newton & Euler equations, it is obvious that we need propagation equations for "$\dot{v} \,\&\, \dot{\omega}$ ".

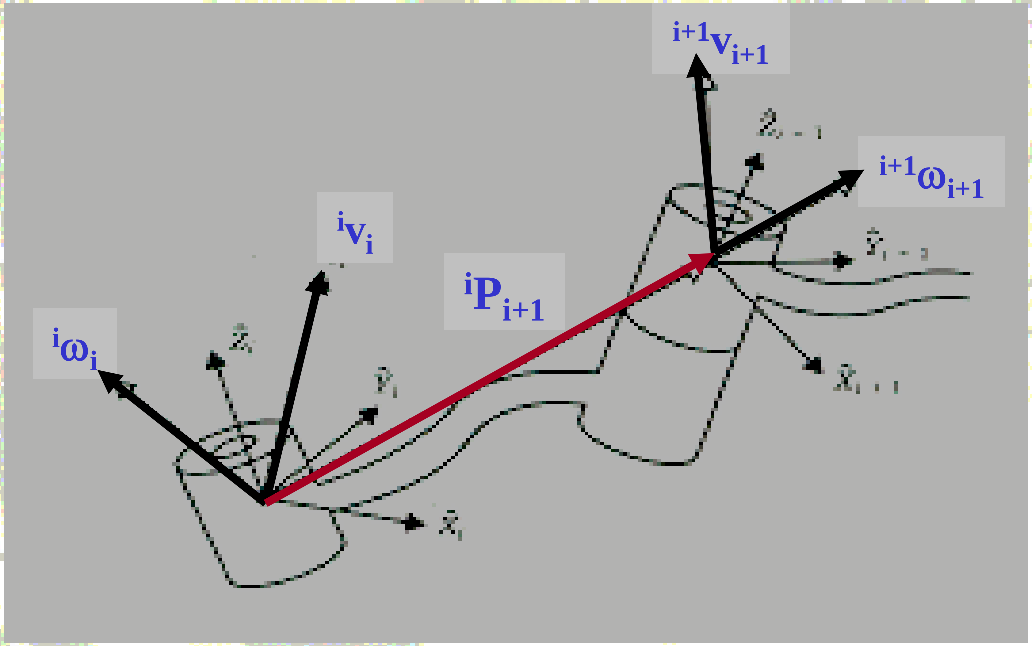
From Chapter-5, the angular velocity equation for every instant is:

$$^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\,{}^{i}\omega_{i} + \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}$$

Differentiating with respect to time we have:

Manipulator Dynamics

$$ {}^{i+1}\dot{\omega}_{i+1} = {}^{i+1}_{i}R\, {}^{i}\dot{\omega}_{i} + {}^{i+1}_{i}R\, {}^{i}\omega_{i} \times \dot{\theta}_{i+1}\, {}^{i+1}\hat{Z}_{i+1} + \ddot{\theta}_{i+1}\, {}^{i+1}\hat{Z}_{i+1} $$



Where:

$$ {}^{i+1}\dot{\hat{Z}}_{i+1} = {}^{i+1}_{i}R\, {}^{i}\omega_{i} \times {}^{i+1}\hat{Z}_{i+1} $$

$$ {}^{i+1}_{i}\dot{R} = {}^{i+1}\omega_{i} \times {}^{i+1}_{i}R \Rightarrow {}^{i+1}_{i}\dot{R}\, {}^{i}\omega_{i} = {}^{i+1}\omega_{i} \times {}^{i+1}_{i}R\, {}^{i}\omega_{i} = {}^{i+1}\omega_{i} \times {}^{i+1}\omega_{i} = 0 $$

Manipulator Dynamics

Also from Chapter-5, the linear velocity equation for every instant is:

$$^{i+1}v_{i+1} = {}^{i+1}_{i}R({}^{i}v_i + {}^{i}\omega_i \times {}^{i}P_{i+1})$$

Differentiating with respect to time we have:

$$^{i+1}\dot{v}_{i+1} = {}^{i+1}_{i}R({}^{i}\dot{v}_i + {}^{i}\dot{\omega}_i \times {}^{i}P_{i+1} + {}^{i}\omega_i \times {}^{i}\dot{P}_{i+1}) \Rightarrow$$

$$^{i+1}\dot{v}_{i+1} = {}^{i+1}_{i}R({}^{i}\dot{v}_i + {}^{i}\dot{\omega}_i \times {}^{i}P_{i+1} + {}^{i}\omega_i \times ({}^{i}\omega_i \times {}^{i}P_{i+1}))$$

Since at every instant:

$$^{i+1}_{i}R = constant \Rightarrow {}^{i+1}_{i}\dot{R} = 0$$

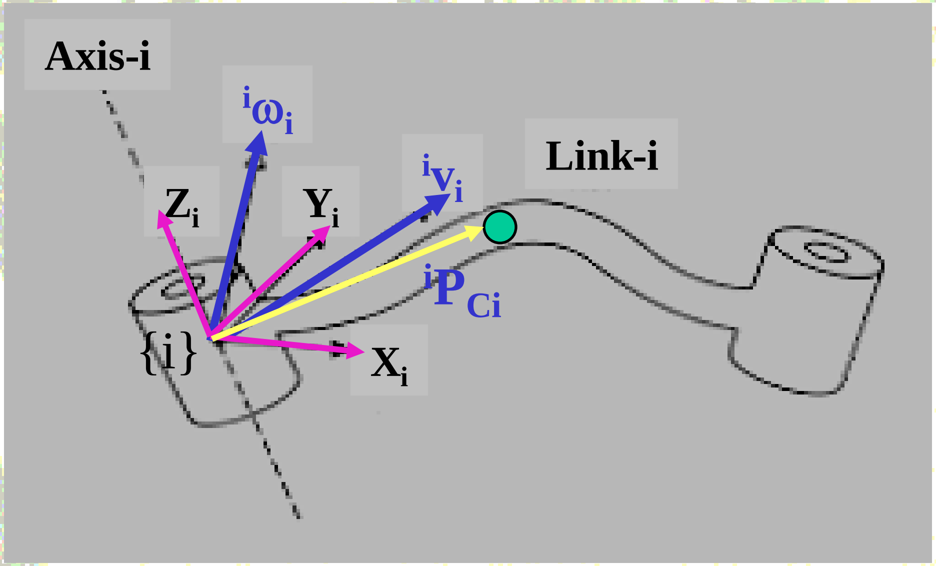
Manipulator Dynamics

To find the linear acceleration of the center of mass, we have:

$$ {}^{i}v_{Ci} = ({}^{i}v_{i} + {}^{i}\omega_{i} \times {}^{i}P_{Ci}) $$

Differentiating with respect to time we have:

$$ {}^{i}\dot{v}_{Ci} = ({}^{i}\dot{v}_{i} + {}^{i}\dot{\omega}_{i} \times {}^{i}P_{Ci} + {}^{i}\omega_{i} \times ({}^{i}\omega_{i} \times {}^{i}P_{Ci}) $$



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Having computed all acceleration equations, we shall now apply the Newton-Euler Equations as follows:

First compute the Inertial Force and Torque acting at the mass center of each link;

$$F_i = m\dot{v}_{Ci} = ma_{Ci}$$

$$N_i = {}^{Ci}I\dot{\omega}_i + \omega_i \times {}^{Ci}I\omega_i$$

${}^{Ci}I$ = Inertia Tensor of the link-i written in frame $\{C_i\}$ with it's origin at the mass center, and having the same orientation as frame {i}.

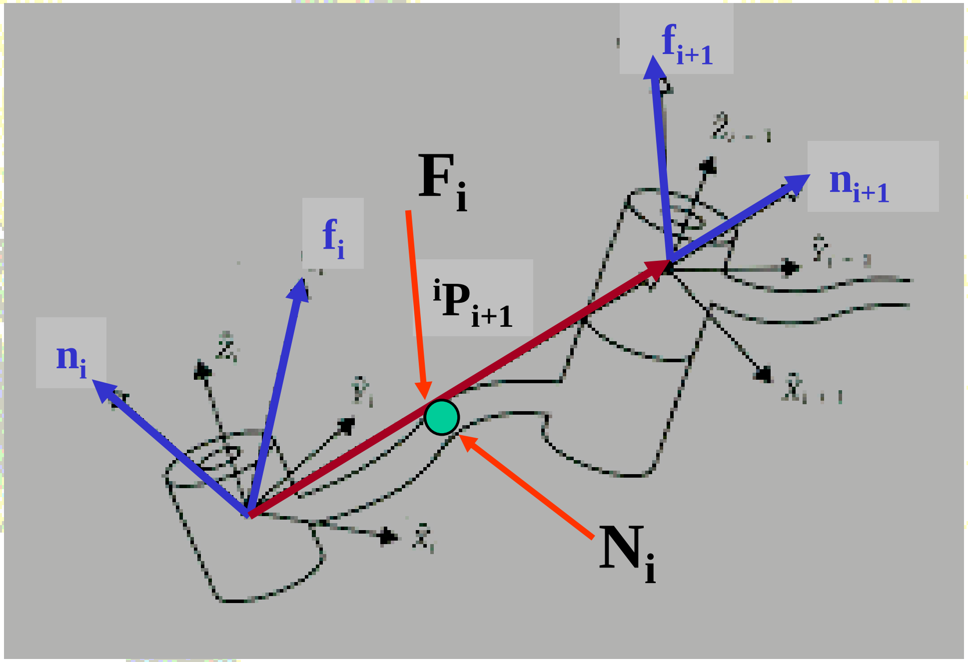
Then, perform Inward Iterations to compute forces and torques;

Manipulator Dynamics

Inward Iterations to Compute Forces and Torques:

Write the force balance on link-i:

$$ {}^{i}f_i = {}^{i}_{i+1}R\, {}^{i+1}f_{i+1} + {}^{i}F_i $$



Write the moment balance about the origin of link frame-i:

$$ {}^{i}n_i = {}^{i}N_i + {}^{i}_{i+1}R\, {}^{i+1}n_{i+1} + {}^{i}P_{Ci} \times {}^{i}F_i + {}^{i}P_{i+1} \times {}^{i}_{i+1}R\, {}^{i+1}f_{i+1} $$

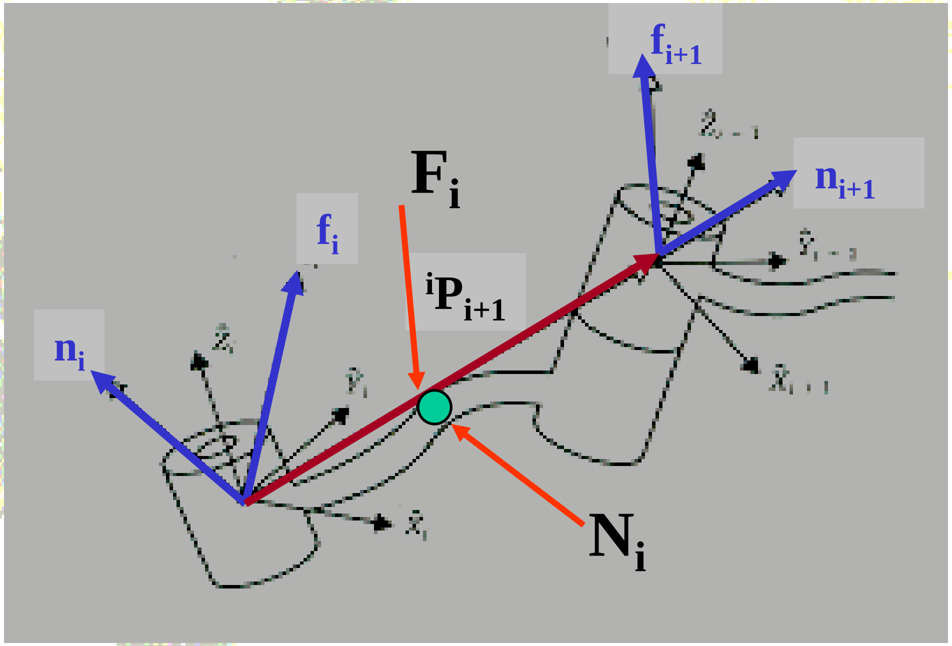
Note: The required joint torques are found by taking the Z-component of the torque applied by one link on it's neighbor.

Manipulator Dynamics

- **Inward Iterations to Compute Forces and Torques:**

Therefore, for Revolute Joints we have:

$$\tau_i = {}^i n_i^T \hat{Z}_i$$



Therefore, for Prismatic Joints we have:

$$\tau_i = {}^i f_i^T \hat{Z}_i$$

Note: For a robot moving in free space, we may have:

$${}^{N+1}f_{N+1} = {}^{N+1}n_{N+1} = 0$$

where as for a robot being in contact with the environment, we may have:

$${}^{N+1}f_{N+1} \neq {}^{N+1}n_{N+1} \neq 0$$

Manipulator Dynamics

➢ Iterative Newton-Euler Dynamic Algorithm:

First: Compute link velocities and accelerations iteratively from link-1 to link-n, and apply the Newton-Euler equations to each link.

Second: Compute the forces and torques of interaction recursively from link-n back to link-1.

Outward iterations: $i : 0 \rightarrow 5$

$$ {}^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\, {}^{i}\omega_i + \dot{\theta}_{i+1}\, {}^{i+1}\hat{Z}_{i+1}, \tag{6.45} $$

$$ {}^{i+1}\dot{\omega}_{i+1} = {}^{i+1}_{i}R\, {}^{i}\dot{\omega}_i + {}^{i+1}_{i}R\, {}^{i}\omega_i \times \dot{\theta}_{i+1}\, {}^{i+1}\hat{Z}_{i+1} + \ddot{\theta}_{i+1}\, {}^{i+1}\hat{Z}_{i+1}, \tag{6.46} $$

$$ {}^{i+1}\dot{v}_{i+1} = {}^{i+1}_{i}R \left({}^{i}\dot{\omega}_i \times {}^{i}P_{i+1} + {}^{i}\omega_i \times \left({}^{i}\omega_i \times {}^{i}P_{i+1} \right) + {}^{i}\dot{v}_i \right), \tag{6.47} $$

$$ \begin{aligned} {}^{i+1}\dot{v}_{C_{i+1}} &= {}^{i+1}\dot{\omega}_{i+1} \times {}^{i+1}P_{C_{i+1}} \\ &\quad + {}^{i+1}\omega_{i+1} \times \left({}^{i+1}\omega_{i+1} \times {}^{i+1}P_{C_{i+1}} \right) + {}^{i+1}\dot{v}_{i+1}, \end{aligned} \tag{6.48} $$

$$ {}^{i+1}F_{i+1} = m_{i+1}\, {}^{i+1}\dot{v}_{C_{i+1}}, \tag{6.49} $$

$$ {}^{i+1}N_{i+1} = {}^{C_{i+1}}I_{i+1}\, {}^{i+1}\dot{\omega}_{i+1} + {}^{i+1}\omega_{i+1} \times {}^{C_{i+1}}I_{i+1}\, {}^{i+1}\omega_{i+1}. \tag{6.50} $$

Inward iterations: $i : 6 \rightarrow 1$

$$ {}^{i}f_i = {}^{i}_{i+1}R\, {}^{i+1}f_{i+1} + {}^{i}F_i, \tag{6.51} $$

$$ \begin{aligned} {}^{i}n_i &= {}^{i}N_i + {}^{i}_{i+1}R\, {}^{i+1}n_{i+1} + {}^{i}P_{C_i} \times {}^{i}F_i \\ &\quad + {}^{i}P_{i+1} \times {}^{i}_{i+1}R\, {}^{i+1}f_{i+1}, \end{aligned} \tag{6.52} $$

$$ \tau_i = {}^{i}n_i^T\, {}^{i}\hat{Z}_i. \tag{6.53} $$

Manipulator Dynamics

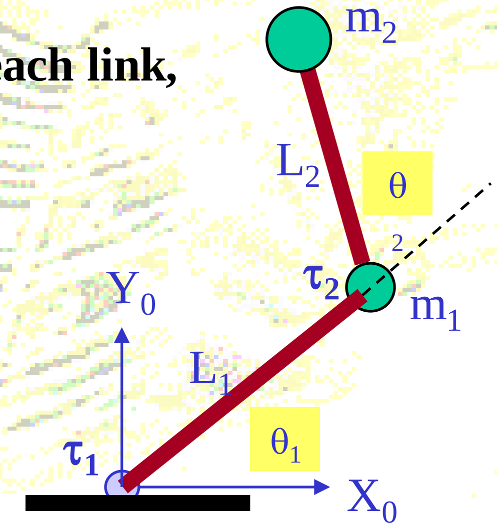
➢ **Closed-form (Symbolic Form) Dynamic Equations:**

Example: The 2-DOF Manipulator Arm.

Assumptions: Point masses at the distal end of each link,

$$^{0}\dot{v}_0 = g\hat{Y}_0 = \begin{bmatrix} 0 \\ g \\ 0 \end{bmatrix}, \qquad (gravity - term)$$

$$\left\{ \begin{matrix} {}^{C1}I_1 = 0 \\ {}^{C2}I_2 = 0 \end{matrix} \right\} (point - mass)$$



$$\tau_1 = m_2\ell_2^2(\ddot{\theta}_1 + \ddot{\theta}_2) + m_2\ell_1\ell_2 C_2(2\ddot{\theta}_1 + \ddot{\theta}_2) + (m_1 + m_2)\ell_1^2\ddot{\theta}_1 - m_2\ell_1\ell_2 S_2\dot{\theta}_2^2 - 2m_2\ell_1\ell_2 S_2\dot{\theta}_1\dot{\theta}_2 + m_2\ell_2 g C_{12} + (m_1 + m_2)\ell_1 g C_1$$

$$\tau_2 = m_2\ell_1\ell_2 C_2\ddot{\theta}_1 + m_2\ell_1\ell_2 S_2\dot{\theta}_1^2 + m_2\ell_2 g C_{12} + m_2\ell_2^2(\ddot{\theta}_1 + \ddot{\theta}_2)$$

Actuator torques as a function of joints position, velocity, and acceleration.